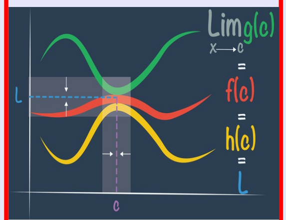
Math 261

Fall 2022

Lecture 12



Class QZ 3

For $\varepsilon>0$, find a $\delta>0$ such that

1) $\lim_{x\to 2}(5x-3)=7$ ✓

$|5x-3-7|<\varepsilon$ whenever $|x-2|<\delta$

$|5x-10|<\varepsilon$ " "

$5|x-2|<\varepsilon$ " "

$|x-2|<\frac{\varepsilon}{5}$ $\boxed{\delta=\frac{\varepsilon}{5}}$

2) $\lim_{x\to 2}(x^2-2x)=0$ ✓

$|x^2-2x-0|<\varepsilon$ whenever $|x-2|<\delta$

$|x(x-2)|<\varepsilon$

$|x|\,|x-2|<\varepsilon$

$\delta\le 1$

$|x-2|<1$

$-1<x-2<1$

$x<3$

$|x|<3$

$|x-2|<\frac{\varepsilon}{3}$

Pick $\delta=\min\{1,\frac{\varepsilon}{3}\}$

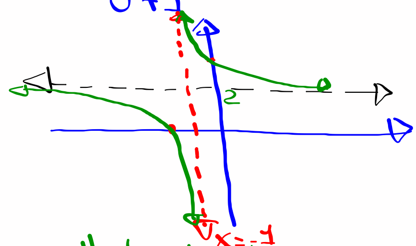
Consider $f(x) = \frac{2x+4}{x+1}$

Domain $x + 1 \neq 0$
$x \neq -1$
V.A.

x-Int $\rightarrow y=0 \rightarrow f(x)=0 \rightarrow \frac{2x+4}{x+1} = 0 \rightarrow \boxed{x=-2}$
$(-2,0)$

Y-Int $\rightarrow x=0 \rightarrow f(0)= \frac{2(0)+4}{0+1} = 4$
$(0,4)$

from pre Calc,
H.A. $\rightarrow y = \frac{2}{1} = 2$



$\lim_{x \to -1^+} f(x) = \infty$

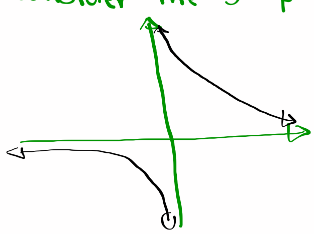
$\lim_{x \to -1^-} f(x) = -\infty$

Now what about limits at $\pm\infty$

$\lim_{x \to \infty} f(x) = 2$

$\lim_{x \to -\infty} f(x) = 2$

Consider the graph of $f(x) = \frac{1}{x}$



$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} \frac{2x+1}{3x-5} = \frac{\infty}{\infty}$ I.F.

$x^1 = x$

Divide everything by the $\boxed{x \text{ to highest power}}$ from numerator & Deno.

$$\lim_{x \to \infty} \frac{2x+1}{3x-5} = \lim_{x \to \infty} \frac{\frac{2x+1}{x}}{\frac{3x-5}{x}} = \lim_{x \to \infty} \frac{\frac{2x}{x} + \frac{1}{x}}{\frac{3x}{x} - \frac{5}{x}}$$

$$= \lim_{x \to \infty} \frac{2 + \frac{1}{x}}{3 - \frac{5}{x}} = \frac{\lim_{x \to \infty}(2 + \cancel{\frac{1}{x}}^{\to 0})}{\lim_{x \to \infty}(3 - \cancel{5/x}^{\to 0})} = \frac{2+0}{3-0}$$

$$= \boxed{\frac{2}{3}}$$

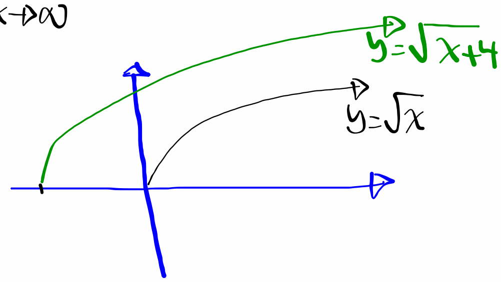
$$\lim_{x\to\infty}\frac{x-4}{x^2+1}=\frac{\infty}{\infty}\ \text{I.F.}$$

It is recommended for $\frac{\infty}{\infty}$ I.F., to divide everything by x highest power from the deno.

$$\lim_{x\to\infty}\frac{x-4}{x^2+1}=\lim_{x\to\infty}\frac{\frac{x}{x^2}-\frac{4}{x^2}}{\frac{\cancel{x^2}}{\cancel{x^2}}+\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\cancel{\frac{1}{x}}^{\to 0}-\cancel{\frac{4}{x^2}}^{\to 0}}{1+\cancel{\frac{1}{x^2}}^{\to 0}}=\frac{0}{1}$$

$$=\boxed{0}$$

$$\lim_{x\to\infty}\left[\sqrt{x+4}-\sqrt{x}\right]=\infty-\infty\ \text{I.F.}$$



$$\lim_{x\to\infty}\left[\sqrt{x+4}-\sqrt{x}\right]=\lim_{x\to\infty}\frac{\left(\sqrt{x+4}-\sqrt{x}\right)\left[\sqrt{x+4}+\sqrt{x}\right]}{\sqrt{x+4}+\sqrt{x}}$$

$$=\lim_{x\to\infty}\frac{4}{\cancel{\sqrt{x+4}+\sqrt{x}}^{\to\infty}}=\frac{4}{\infty}$$

$$=\boxed{0}$$

Evaluate $\lim\limits_{x\to\infty}\left[\sqrt{x^2+4x}-x\right] = \infty-\infty$ I.F.

$$\lim_{x\to\infty}\left(\sqrt{x^2+4x}-x\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+4x}-x\right)\left(\sqrt{x^2+4x}+x\right)}{\sqrt{x^2+4x}+x}$$

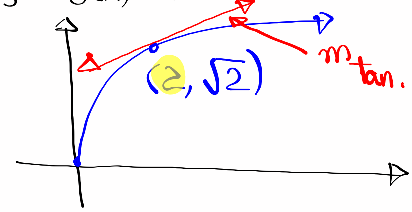
$$=\lim_{x\to\infty}\frac{x^2+4x-x^2}{\sqrt{x^2+4x}+x}=\lim_{x\to\infty}\frac{4x}{\sqrt{x^2+4x}+x}\quad \frac{\infty}{\infty}\ \text{I.F.}$$

$$=\lim_{x\to\infty}\frac{\frac{4x}{x}}{\frac{\sqrt{x^2+4x}}{x}+\frac{x}{x}}=\lim_{x\to\infty}\frac{4}{\sqrt{\frac{x^2+4x}{x^2}}+1}$$

$$=\lim_{x\to\infty}\frac{4}{\sqrt{1+\cancel{\frac{4}{x}}^{\,0}}+1}$$

$$=\frac{4}{1+1}=\frac{4}{2}=\boxed{2}$$

Find slope of the tan. line to the graph of $f(x)=\sqrt{x}$ at $x=2$.



$$m_{\text{tan. line}}=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to0}\frac{f(2+h)-f(2)}{h}$$

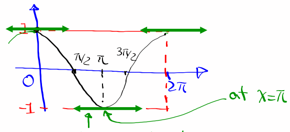
$$=\lim_{h\to0}\frac{\sqrt{2+h}-\sqrt{2}}{h}=\frac{0}{0}\ \text{I.F.}$$

$$=\lim_{h\to0}\frac{\left(\sqrt{2+h}-\sqrt{2}\right)\left(\sqrt{2+h}+\sqrt{2}\right)}{h\left(\sqrt{2+h}+\sqrt{2}\right)}=\lim_{h\to0}\frac{\cancel{2}+\cancel{h}^{\,1}-\cancel{2}}{\cancel{h}\left(\sqrt{2+h}+\sqrt{2}\right)}$$

$$=\lim_{h\to0}\frac{1}{\sqrt{2+h}+\sqrt{2}}=\frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\boxed{\frac{\sqrt{2}}{4}}$$

Consider the graph of $f(x) = \cos x$ on $[0, 2\pi]$

find all points on the graph where tan. line is horizontal.



Horizontal tan. line $\Rightarrow m = 0$

$$m_{\text{tan. line}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \frac{0}{0}$$

I.F.

$$= \lim_{h \to 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$$

Recall
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \lim_{h \to 0} \frac{\cos x [\cosh - 1] - \sin x \sinh}{h}$$

$\lim_{h \to 0} \frac{\cosh - 1}{h} = 0$

$\lim_{h \to 0} \frac{\sinh}{h} = 1$

$$= \lim_{h \to 0} \frac{\cos x [\cosh - 1]}{h} - \lim_{h \to 0} \frac{\sin x \sinh}{h}$$

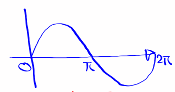
$$= \cos x \underbrace{\lim_{h \to 0} \frac{\cosh - 1}{h}}_{0} - \sin x \underbrace{\lim_{h \to 0} \frac{\sinh}{h}}_{1}$$

$$= -\sin x$$

Slope of tan. line at any Point on the graph of $f(x) = \cos x$

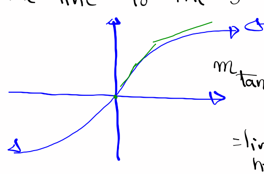
$m = 0$ $\qquad -\sin x = 0 \qquad \sin x = 0$

Horizontal tan. line



$x = 0 \qquad \boxed{x = \pi} \qquad x = 2\pi$

Find a general form for the slope of the line to the graph of $f(x) = \sqrt[3]{x}$



$$m_{\text{tan. line}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} = \frac{0}{0}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt[3]{x+h} - \sqrt[3]{x}\right)\left(\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2}\right)}{h\left(\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2}\right)}$$

Recall
$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$$= \lim_{h \to 0} \frac{\left(\sqrt[3]{x+h}\right)^3 - \left(\sqrt[3]{x}\right)^3}{h\left[\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2}\right]}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h}^{\,1} - \cancel{x}}{\cancel{h}\left(\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2}\right)} = \frac{1}{\sqrt[3]{x^2} + \sqrt[3]{x^2} + \sqrt[3]{x^2}}$$

$$= \frac{1}{3\sqrt[3]{x^2}}$$

Slope of tan. line at any Point on the graph of $f(x) = \sqrt[3]{x}$

what happens at $x = 0$? We have a Vertical Tangent line.